Date

MATH2230 Complex Variables with Application
Suggested Solution for HW1

Ch1, SEC.3 Exercises

1. (a) $\dfrac{1+2i}{3-4i}+\dfrac{2-i}{5i} = \dfrac{(1+2i)(3+4i)}{(3-4i)(3+4i)}+\dfrac{(2-i)5i}{(5i)^2} = \dfrac{-5+10i}{25}+\dfrac{10i+5}{-25} = -\dfrac{2}{5}$

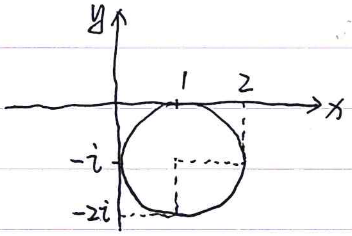
   (b) $\dfrac{5i}{(1-i)(2-i)(3-i)} = \dfrac{5i(1+i)(2+i)(3+i)}{2\times 5\times 10} = \dfrac{(5i-5)(5+5i)}{100} = -\dfrac{1}{2}$

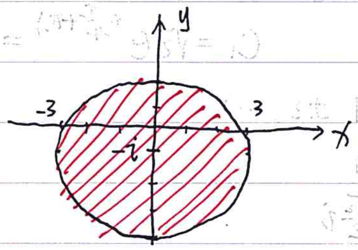
   (c) $(1-i)^4 = (-2i)^2 = -4$

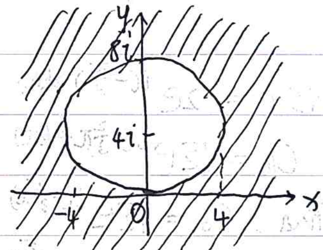
5. $\dfrac{z_1}{z_2} = \dfrac{(x_1+iy_1)(x_2-iy_2)}{(x_2+iy_2)(x_2-iy_2)} = \dfrac{x_1x_2+y_1y_2}{x_2^2+y_2^2}+i\dfrac{y_1x_2-x_1y_2}{x_2^2+y_2^2}$ $(z_2\neq 0)$

Ch1, SEC.5 Exercises

5. (a) $|z-1+i| = 1$ (b) $|z+i|\leq 3$ (c) $|z-4i|\geq 4$







6. Remark: Nomatter what you argued, you will obtain full marks as long as you reasoned correct

   Suggested solution: $|x+iy-1| = |x+iy+i|$

   $(x-1)^2+y^2 = x^2+(y+1)^2$

   $-x = y$

   which represents a line through the origin whose slope is -1

8. Proof: $|(x_1+iy_1)(x_2+iy_2)| = |(x_1x_2-y_1y_2)+i(x_1y_2+x_2y_1)|$

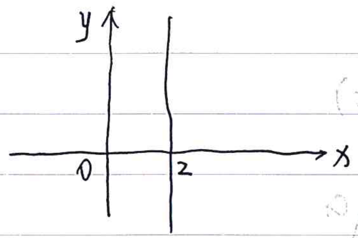
   $= \sqrt{(x_1x_2-y_1y_2)^2+(x_1y_2+x_2y_1)^2} = \sqrt{(x_1^2+y_1^2)(x_2^2+y_2^2)}$

   which implies $|z_1z_2| = |z_1||z_2|$

Ch1, SEC.6 Exercises

2. (a) $\mathrm{Re}(\bar{z}-i) = 2$

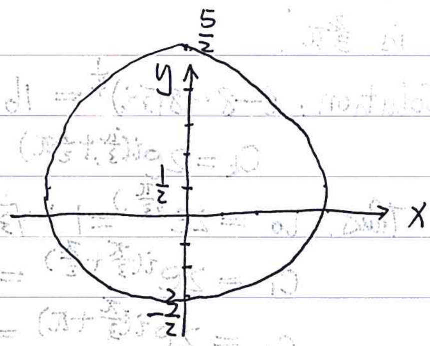
   i.e $\mathrm{Re}(x-iy-i) = 2$



   (b) $|2\bar{z}+i| = 4$

   i.e. $|2x+(1-2y)i| = 4$

   $x^2+(y-\frac{1}{2})^2 = 4$



13. Proof: Since $|z-z_0|=R$, we have $|z-z_0|^2=R^2$

i.e. $(z-z_0)\overline{(z-z_0)}=R^2$

$$(z-z_0)(\bar{z}-\bar{z}_0)=R^2$$

$$z\bar{z}-z_0\bar{z}-z\bar{z}_0+z_0\bar{z}_0=R^2$$

$$|z|^2-z_0\bar{z}-\overline{z_0\bar{z}}+|z_0|^2=R^2$$

$$|z|^2-2\mathrm{Re}(z\bar{z}_0)+|z_0|^2=R^2$$

Ch1, SEC.11 Exercises

1. Solution: (a) $2i=2e^{i(\frac{\pi}{2}+2k\pi)}$ $(k=0,\pm1,\pm2,\cdots)$

$C_k=\sqrt{2}e^{i(\frac{\pi}{4}+k\pi)}$ $(k=0,1)$

Thus, the square roots of $2i$ are $C_0=\sqrt{2}e^{i\frac{\pi}{4}}=1+i$

$C_1=\sqrt{2}e^{i(\frac{\pi}{4}+\pi)}=-1-i$

(b) $1-\sqrt{3}i=2e^{i(-\frac{\pi}{3}+2k\pi)}$ $(k=0,\pm1,\pm2,\cdots)$

$C_k=\sqrt{2}e^{i(-\frac{\pi}{6}+k\pi)}$ $(k=0,1)$

Thus, $C_0=\sqrt{2}e^{i(-\frac{\pi}{6})}=\frac{\sqrt{3}}{\sqrt{2}}-\frac{\sqrt{2}}{2}i$

$C_1=\sqrt{2}e^{i(-\frac{\pi}{6}+\pi)}=-\frac{\sqrt{3}}{\sqrt{2}}+\frac{\sqrt{2}}{2}i$

2. Solution: $-8i=8e^{i(-\frac{\pi}{2}+2k\pi)}$ $(k=0,\pm1,\pm2,\cdots)$

$C_k=2e^{i(-\frac{\pi}{6}+\frac{2}{3}k\pi)}$ $(k=0,1,2)$

Thus $C_0=2e^{i(-\frac{\pi}{6})}=\sqrt{3}-i$

$C_1=2e^{i(-\frac{\pi}{6}+\frac{2}{3}\pi)}=2i$

$C_2=2e^{i(-\frac{\pi}{6}+\frac{4}{3}\pi)}=-\sqrt{3}-i$

Note that the three cube roots $C_k(k=0,1,2)$ of $-8i$ can be written

$C_0, C_0\omega_3, C_0\omega_3^2$ where $\omega_3=e^{i\frac{2}{3}\pi}$

which implies the three roots have the same modulus 2 and the angle between each of them is $\frac{2}{3}\pi$.

3. Solution: $-8-8\sqrt{3}i=16e^{i(-\frac{2}{3}\pi+2k\pi)}$ $(k=0,\pm1,\pm2,\cdots)$

$C_k=2e^{i(-\frac{\pi}{6}+\frac{k}{2}\pi)}$ $(k=0,1,2,3)$

Thus, $C_0=2e^{i(-\frac{\pi}{6})}=\sqrt{3}-i$ (C_0 is the principal root)

$C_1=2e^{i(-\frac{\pi}{6}+\frac{\pi}{2})}=1+\sqrt{3}i$

$C_2=2e^{i(-\frac{\pi}{6}+\pi)}=-\sqrt{3}+i$

$C_3=2e^{i(-\frac{\pi}{6}+\frac{3}{2}\pi)}=-1-\sqrt{3}i$